4.D: the ***PhilosophyIdeas*** overview of

Modal Logic

Once a logic was established for propositions (typically connecting them with 'and', 'not, 'or' and 'if…then'), and then a logic was developed for predicates (typically using two types of variable, and the quantifiers 'for all' and 'there is' to control them), further areas of reasoning invited investigation, and the logic of 'necessarily' and 'possibly' looked the most philosophically interesting. One simple step opened up the field, of introducing an operator for 'necessary' (written $\Box$, called '**box**'), and an operator for 'possibly' (written $\Diamond$, called '**diamond**'). If you have one you can define the other, because necessity is not-possibly-false, and possibility is not-necessarily-false.

Given this starting point, it became obvious that there is no single system of modal logic, because you have to decide what basic implications are accepted. If a truth is necessary, must there be an instance of that truth? Are possibility and necessity absolute or relative notions? If a truth is necessary, might it have been contingent? There are many such questions, and different answers implied different basic assumptions. The problem is to decide exactly what we mean by 'necessary' and 'possible' (and that problem is much clearer when modal logic is used). We will outline six of the systems, keeping symbols to a minimum (though a good analytic philosopher can't avoid them!).

Standard predicate logic is used, but the systems are defined by their axioms. These can be mere formal syntax, but the systems are clearer if we outline the **semantics** of possible worlds, which gives meaning to the systems by assigning truth-values to its propositions. The key idea is to imagine a set of possible worlds, and then say that if proposition p is true in all of them then it is 'necessary', because it cannot fail to be true, and that if p is true in any one them then it is 'possible', because that shows it can be true.

The new idea for the semantics is to consider whether worlds are '**accessible**' from one another (or one world can '**see**' another), meaning that from one world the truth of some proposition in another world can be known. So if I say 'donkeys might talk', I know 'donkeys talk' is false in this world, but I can 'see' a possible world where it is true. A 'frame' is a system of possible worlds, with an accessibility relation defined for the system, using these relations:

- **Serial** – every world can always see at least one other world
- **Reflexive** – every world can see itself
- **Symmetric** – any pair of worlds can see one another
- **Transitive** – for three worlds, if the first sees the second, and the second the third, the first sees the third

Each of these relations implies a 'hallmark' axiom, a truth that results from the relation. **Serial** means that being necessary implies being possible ($\Box p \rightarrow \Diamond p$), because being serial means a world can be seen, and $\Box p$ means p is true in that world, so it passes the test for $\Diamond p$. **Reflexive** means that being necessary implies that it is true ($\Box p \rightarrow p$), because $\Box p$ makes it true in its own world, and that can be seen. **Symmetric** means that p being true implies that p is necessarily possible ($p \rightarrow \Box\Diamond p$), because the second world must be able to see that p is possible in the first world. **Transitive** means that being necessarily true makes it necessary that it is necessarily true ($\Box p \rightarrow \Box\Box p$), because every world can see $\Box p$ (and not just p). The main systems, with their relations and a hallmark axiom, are:

- **K** – the basic system, with no accessibility conditions enforced [$\Box(p \wedge q) \rightarrow (\Box p \wedge \Box q)$, true in all systems]
- **D** – accessibility is serial [$\Box p \rightarrow \Diamond p$]
- **T** – accessibility is serial and reflexive [$\Box p \rightarrow p$]
- **B** – accessibility is serial, reflexive and symmetric [$p \rightarrow \Box\Diamond p$]
- **S4** – accessibility is serial, reflexive and transitive [$\Box p \rightarrow \Box\Box p$]
- **S5** – accessibility is serial, reflexive, transitive and symmetric [$\Diamond p \rightarrow \Box\Diamond p$]

The systems increase in strength, adding previous axioms, and further modal truths emerge. **S4** lacks symmetry which means 'you can get there but can't get back' in a chain of inferences; so that the starting point (the actual world) has a special place in the system. By contrast, in **S5** every world can see every other world, so accessibility becomes irrelevant; this means that S5 offers the big overview of things, and is usually said to be the logic of metaphysics. In S5 if anything is possible, or anything is necessary, then that is necessarily so, rather than being relative to some viewpoint. Hence S5 seems to describe 'absolute' necessities and possibilities. S5 also has the interesting $\Diamond\Box p \rightarrow \Box p$ as a theorem ('if it *could be* necessarily true, then it *is* necessarily true'), because $\Diamond\Box p$ means $\Box p$ is true in some world, but in S5 that makes p true and seen to be true in every world, and hence $\Box p$ is true in all worlds (so in S5 if God could have necessary existence, it follows that God does necessarily exist).

By interpreting $\Box$ as 'always' and $\Diamond$ as 'sometimes', these systems can be used for **Temporal Logics**. System **T** seems to suit this, because temporal ordering (before/after) is asymmetric, and 'always' implies 'sometimes'. **Deontic Logics** aims to reason about moral duties, by thinking of $\Box$ as 'obligatory' and $\Diamond$ as 'permissible'. System **D** seems suitable here, because ought ($\Box p$) implies can ($\Diamond p$) , but ought does not imply that it happens (p); the latter would need system T. There are also **Epistemic Logics**, treating $\Box$ as 'knows' and $\Diamond$ as 'seems possible'.

The semantics of predicate modal logic has a problem with referring to objects. Does 'you might have been taller' refer to you, if the possible person lacks your actual height? To deal with this, the idea was introduced of 'rigidly' referring to you, meaning it is you even if your height is different, because your height is not one of your necessary features. This means there are necessities about things (*de re*), as well as necessarily true sentences (*de dicto*).

Big claims about necessity talk of being true 'in all possible worlds', and usually invoke S5, but natural necessities seem to depend on a context (such as necessarily having a certain weight, but only on this planet). Sceptics say that all necessities depend on a context, or a mode of description, and so retreat from S5, or give up modal logic entirely. Another interesting issue is whether there are possible objects, and whether mere possibilities are truths in this world. We must decide whether possible objects are available for all worlds, or are restricted to 'local domains'.